Mathématiques E F G – Corrigé

I **1)** $A(2\ ;\ -1\ ;\ 5)$, $B(3\ ;\ 4\ ;\ -5)$ et $C(3\ ;\ -1\ ;\ 0)$.

a) $M(x\ ;\ y\ ;\ z) \in (AB) \Leftrightarrow$ il existe $k \in \mathbb{R}$ tel que $\overrightarrow{AM} = k \cdot \overrightarrow{AB}$ ($k \in \mathbb{R}$)

Or : $\overrightarrow{AM} = k \cdot \overrightarrow{AB} \Leftrightarrow \begin{cases} x-2=k \\ y+1=5k \\ z-5=-10k \end{cases} \Leftrightarrow \boxed{\begin{cases} x=2+k & (1) \\ y=-1+5k & (2) \\ z=5-10k & (3) \end{cases}}$

(1) dans (2) et (3) : $\begin{cases} y+1=5(x-2) \\ z-5=-10(x-2) \end{cases} \Leftrightarrow \begin{cases} y+1=5x-10 \\ z-5=-10x+20 \end{cases} \Leftrightarrow \boxed{\begin{cases} -5x+y+11=0 \\ 10x+z-25=0 \end{cases}}$

b) $M(x\ ;\ y\ ;\ z) \in (ABC) \Leftrightarrow$ il existe $\alpha, \beta \in \mathbb{R}$ tels que $\overrightarrow{AM} = \alpha \cdot \overrightarrow{AB} + \beta \cdot \overrightarrow{AC}$ ($\alpha, \beta \in \mathbb{R}$)

Or : $\overrightarrow{AM} = \alpha \cdot \overrightarrow{AB} + \beta \cdot \overrightarrow{AC} \Leftrightarrow \begin{cases} x-2=\alpha+\beta \\ y+1=5\alpha \\ z-5=-10\alpha-5\beta \end{cases} \Leftrightarrow \boxed{\begin{cases} x=2+\alpha+\beta \\ y=-1+5\alpha \\ z=5-10\alpha-5\beta \end{cases}}$

$$\begin{cases} x=2+\alpha+\beta \\ y=-1+5\alpha \\ z=5-10\alpha-5\beta \end{cases} \Leftrightarrow \begin{cases} \beta=x-2-\alpha \\ \alpha=\frac{y+1}{5} \\ z=5-10\alpha-5\beta \end{cases} \Leftrightarrow \begin{cases} \beta=x-2-\frac{y+1}{5} \\ \alpha=\frac{y+1}{5} \\ z=5-10\cdot\frac{y+1}{5}-5\cdot\left(x-2-\frac{y+1}{5}\right) \quad (*) \end{cases}$$

$(*) \Leftrightarrow z=5-2y-2-5x+10+y+1 \Leftrightarrow \boxed{5x+y+z-14=0}$

2) $\begin{cases} x-y-2z=-3 \\ 5x-2y-2z=-1 \\ 4x+2y+z=2 \end{cases} \Leftrightarrow \begin{cases} x-y-2z=-3 \\ 4x-y=2 \\ 9x+3y=1 \end{cases} \Leftrightarrow \begin{cases} x-y-2z=-3 \\ 4x-y=2 \\ 21x=7 \end{cases} \Leftrightarrow \begin{cases} x=\frac{1}{3} \\ y=-\frac{2}{3} \\ z=2 \end{cases}$ $S=\{(\frac{1}{3}\ ;-\frac{2}{3}\ ;\ 2)\}$

<u>Interprétation géométrique :</u>

Les trois plans qui correspondent aux trois équations se coupent au point $I(\frac{1}{3}\ ;-\frac{2}{3}\ ;\ 2)$.

II **1)** **a)** $C_{13}^2 \cdot C_{17}^2 = 78 \cdot 136 = 10608$

b) $C_{13}^4 + C_{17}^4 = 715 + 2380 = 3095$

2) **a)** $15^3 = 3375$

b) $4^3 + 5^3 + 6^3 = 405$

c) $3 \cdot (4 \cdot 4 \cdot 6) = 3 \cdot 96 = 288$

III 1) $\boxed{2\ln(x+7) = \ln(-x-3) + \ln(-2x-8)}$

C.E. : $x+7>0 \Leftrightarrow x>-7$
$-x-3>0 \Leftrightarrow x<-3$
$-2x-8>0 \Leftrightarrow x<-4$

$D =]-7\ ;\ -4[$

$\forall\ x \in D$: $2\ln(x+7) = \ln(-x-3) + \ln(-2x-8)$

$\Leftrightarrow \ln[(x+7)^2] = \ln(-x-3)(-2x-8)$

$\Leftrightarrow x^2 + 14x + 49 = 2x^2 + 8x + 6x + 24$

$\Leftrightarrow -x^2 + 25 = 0$

$\Leftrightarrow x^2 - 25 = 0$

$\Leftrightarrow x = 5$ ou $x = -5$ $\qquad S = \{-5\}$

2) $\boxed{e^{x(x-2)} \geq e^x \cdot \left(e^{x-3}\right)^2}$

$\forall\ x \in \mathbb{R}$: $e^{x(x-2)} \geq e^x \cdot \left(e^{x-3}\right)^2$

$\Leftrightarrow e^{x^2-2x} \geq e^{x+2x-6}$

$\Leftrightarrow x^2 - 2x \geq 3x - 6$

$\Leftrightarrow x^2 - 5x + 6 \geq 0$ $\qquad [\Delta = 1]$

$\Leftrightarrow x \leq 2$ ou $x \geq 3$ $\qquad S =]-\infty\ ;\ 2] \cup [3\ ;\ +\infty[$

IV 1) $\boxed{f(x) = \ln\left(\frac{2x-3}{3x-2}\right)}$

C.E. : $3x-2 \neq 0 \Leftrightarrow x \neq \frac{2}{3}$

$\frac{2x-3}{3x-2} > 0$

x	$-\infty$		$\frac{2}{3}$		$\frac{3}{2}$		$+\infty$
$2x-3$		$-$		$-$	0	$+$	
$3x-2$		$-$	0	$+$		$+$	
$\frac{2x-3}{3x-2}$		$+$	‖	$-$	0	$+$	

$dom\ f =]-\infty\ ;\ \frac{2}{3}[\cup]\frac{3}{2}\ ;\ +\infty[$

$$f'(x) = \frac{1}{\frac{2x-3}{3x-2}} \cdot \frac{2(3x-2)-(2x-3)\cdot 3}{(3x-2)^2} = \frac{3x-2}{2x-3} \cdot \frac{6x-4-6x+9}{(3x-2)^2} = \frac{3x-2}{2x-3} \cdot \frac{5}{(3x-2)^2} = \underline{\underline{\frac{5}{(2x-3)(3x-2)}}}$$

2) $\boxed{f(x) = \frac{2+e^{3x}}{2-e^{3x}}}$

C.E. : $2 - e^{3x} \neq 0 \Leftrightarrow e^{3x} \neq 2 \Leftrightarrow 3x \neq \ln 2 \Leftrightarrow x \neq \frac{1}{3}\ln 2$

$dom\ f = \mathbb{R} \setminus \{ \frac{1}{3}\ln 2 \}$

$$f'(x) = \frac{3e^{3x}(2-e^{3x})-(2+e^{3x})\cdot(-3e^{3x})}{(2-e^{3x})^2} = \frac{6e^{3x}-3e^{6x}+6e^{3x}+3e^{6x}}{(2-e^{3x})^2} = \underline{\underline{\frac{12e^{3x}}{(2-e^{3x})^2}}}$$

V **1)** $\int_{-1}^{0} \frac{3}{(2x-1)^3}\,dx = \int_{-1}^{0} 3(2x-1)^{-3}\,dx$

$$= \left[\frac{3}{2}\cdot\frac{(2x-1)^{-2}}{-2}\right]_{-1}^{0}$$

$$= \left[-\frac{3}{4(2x-1)^2}\right]_{-1}^{0}$$

$$= (-\tfrac{3}{4}) - (-\tfrac{1}{12})$$

$$= -\tfrac{9}{12} + \tfrac{1}{12}$$

$$= \underline{\underline{-\tfrac{2}{3}}}$$

2) $\int \underbrace{(2x-1)}_{f}\underbrace{e^{2x}}_{g'}dx = \underbrace{(2x-1)}_{f}\underbrace{\tfrac{1}{2}e^{2x}}_{g} - \int \underbrace{2}_{f'}\underbrace{\tfrac{1}{2}e^{2x}}_{g}dx$

$$= (x-\tfrac{1}{2})e^{2x} - \int e^{2x}dx$$

$$= (x-\tfrac{1}{2})e^{2x} - \tfrac{1}{2}e^{2x} + c \quad (c \in \mathbb{R})$$

$$= \underline{\underline{(x-1)e^{2x} + c}}$$

VI $\int_{1}^{5}\left[f(x)-g(x)\right]dx = \int_{1}^{5}(-\tfrac{6}{5}x + \tfrac{36}{5} - \tfrac{6}{x})\,dx$

$$= \left[-\frac{6}{5}\cdot\frac{x^2}{2} + \frac{36}{5}x - 6\ln|x|\right]_{1}^{5}$$

$$= \left[-\tfrac{3}{5}x^2 + \tfrac{36}{5}x - 6\ln|x|\right]_{1}^{5}$$

$$= (-15 + 36 - 6\ln 5) - (-\tfrac{3}{5} + \tfrac{36}{5} - 0)$$

$$= 21 - 6\ln 5 - \tfrac{33}{5}$$

$$= \tfrac{72}{5} - 6\ln 5$$

$$\approx 4{,}74\,u.a.$$
