I) 1) voir livre page

2) $$\int \frac{6x-1}{\sqrt{4-x^2}}\,dx = \int \frac{6x}{\sqrt{4-x^2}}\,dx - \int \frac{1}{\sqrt{4-x^2}}\,dx$$

Poser $u(x) = 4-x^2$.
Alors $u'(x) = -2x$

$$= -3 \int \underbrace{-2x}_{u'(x)} \underbrace{(4-x^2)}_{u(x)}^{-1/2}\,dx - \int \frac{1}{2\sqrt{1-(\frac{x}{2})^2}}\,dx$$

$$= -3\cdot 2(4-x^2)^{1/2} - \int \frac{1/2}{\sqrt{1-(\frac{x}{2})^2}}\,dx$$

Poser $v(x) = \frac{x}{2}$.
Alors $v'(x) = \frac{1}{2}$.

$$= -6\sqrt{4-x^2} - \operatorname{Arcsin}\frac{x}{2} + k$$

$$\int_0^{\sqrt{3}} \frac{6x-1}{\sqrt{4-x^2}}\,dx = -6\cdot 1 - \underbrace{\operatorname{Arcsin}\frac{\sqrt{3}}{2}}_{=\frac{\pi}{3}} + 6\cdot 2 + \underbrace{\operatorname{Arcsin} 0}_{=0}$$

$$= 6 - \frac{\pi}{3}$$

2) b) $$\int \sin x\, e^{\cos x}\,dx$$

Poser $u(x) = \cos x$.
Alors $u'(x) = -\sin x$

$$= -\int \underbrace{-\sin x}_{u'(x)} \cdot e^{\overset{u(x)}{\cos x}}\,dx = -e^{\cos x} + k$$

$$\int \sin x \cdot \cos x\, e^{\cos x}\,dx$$

IPP avec
$u(x) = \cos x$ et $v'(x) = \sin x\, e^{\cos x}$
$\to u'(x) = -\sin x$ et $v(x) = -e^{\cos x}$

$$= -\cos x \cdot e^{\cos x} - \int \sin x\, e^{\cos x}\,dx$$

$$= -\cos x\, e^{\cos x} + e^{\cos x} + k$$

II) 1) $$\log_{\sqrt{2}} 4 - \log 0{,}1 + 3^{3\log_3 2} - e^{-\ln 2}$$

$$= \log_{\sqrt{2}} \sqrt{2}^4 - \log_{10} 10^{-1} + 3^{\log_3 2^3} - e^{\ln 2^{-1}}$$

$$= 4 - (-1) + 2^3 - 2^{-1} = \frac{25}{2}$$

2) $6^{x} - 6^{1-x} = 5$ CE: –

$\Leftrightarrow 6^{x} - 6 \cdot 6^{-x} = 5 \quad | \cdot 6^{x} \quad \Leftrightarrow 6^{2x} - 6 = 5 \cdot 6^{x}$. Poser $t = 6^{x}$ Alors $t > 0$.

$\Leftrightarrow t^{2} - 5t - 6 = 0 \qquad \Delta = 25 + 24 = 49 > 0$

$\Leftrightarrow \underbrace{t = \frac{5-7}{2} = -1}_{\text{à écarter}}$ ou $t = \frac{5+7}{2} = 6$

$\Leftrightarrow 6^{x} = 6 \quad \Leftrightarrow \quad x = 1 \qquad S = \{1\}$

---

3) $4 \cdot \log_{1/4}(3-x) + \log_{2}(2x+6) \leq 1 \qquad (I)$

CE: $\begin{cases} 3 - x > 0 \\ 2x + 6 > 0 \end{cases} \Leftrightarrow \begin{cases} x < 3 \\ x > -3 \end{cases} \Leftrightarrow x \in \,]-3, 3[$

$(I) \Leftrightarrow 4 \cdot \frac{\ln(3-x)}{\ln 1/4} + \frac{\ln(2x+6)}{\ln 2} \leq 1$

$\Leftrightarrow \frac{\cancel{4}^{2} \ln(3-x)}{-\cancel{2} \ln 2} + \frac{\ln(2x+6)}{\ln 2} \leq 1 \quad | \cdot \underbrace{\ln 2}_{>0}$

$\Leftrightarrow -2 \ln(3-x) + \ln(2x+6) \leq \ln 2$

$\Leftrightarrow \ln(2x+6) \leq \ln 2 + 2 \ln(3-x)$

$\Leftrightarrow \ln(2x+6) \leq \ln[2 \cdot (3-x)^{2}]$

$\Leftrightarrow 2x + 6 \leq 2(9 - 6x + x^{2})$

$\Leftrightarrow 2x^{2} - 14x + 12 \geq 0 \quad | : 2$

$\Leftrightarrow x^{2} - 7x + 6 \geq 0$

$\Leftrightarrow x \leq 1$ ou $x \geq 6$

$\Delta = 49 - 24 = 25 > 0$

Racines de $x^{2} - 7x + 6$:

$x_{1} = \frac{7-5}{2} = 1$ et $x_{2} = \frac{7+5}{2} = 6$

$S = \,]-3, 1]$

x		1		6	
$x^{2}-7x+6$	+	0	−	0	+

---

4) $f(x) = \left(\frac{2x+1}{2x}\right)^{x/2} = e^{\ln\left(\frac{2x+1}{2x}\right)^{x/2}} = e^{\frac{x}{2} \cdot \ln \frac{2x+1}{2x}}$

$\lim\limits_{x \to +\infty} f(x) = e^{\lim\limits_{x \to +\infty} \underbrace{\frac{x}{2}}_{\to +\infty} \cdot \underbrace{\ln \frac{2x+1}{2x}}_{\to 0}}$ $\quad \frac{2x+1}{2x} \to 1$ $\qquad$ (fi : $+\infty \cdot 0$)

$$\lim_{x\to+\infty} \frac{x}{2}\cdot \ln\frac{2x+1}{2x} = \lim_{x\to+\infty} \frac{\ln\frac{2x+1}{2x} \;)\to 0}{\frac{2}{x}\;)\to 0} \quad \text{fi } \frac{0}{0}$$

$$\overset{(H)}{=} \lim_{x\to+\infty} \frac{\dfrac{\left(\frac{2x+1}{2x}\right)'}{\frac{2x+1}{2x}}}{-\frac{2}{x^2}} \qquad \left(\frac{2x+1}{2x}\right)' = \frac{2x\cdot 2-(2x+1)\cdot 2}{4x^2} = -\frac{1}{2x^2}$$

$$= \lim_{x\to+\infty} \frac{-\frac{1}{2x^2}\cdot\frac{2x}{2x+1}}{-\frac{2}{x^2}} = \lim_{x\to+\infty} \frac{x}{2(2x+1)} = \lim_{x\to+\infty}\frac{x}{4x} = \frac{1}{4}$$

Donc $\lim_{x\to+\infty} f(x) = e^{1/4} = \sqrt[4]{e}$.

III)

$$f(x) = -\frac{x}{2} + \ln\frac{x-1}{x}$$

1) Etude de f

a) $\mathrm{Dom}\, f =]-\infty, 0[\cup]1,+\infty[$

CE: $\begin{cases} x \neq 0 \\ \frac{x-1}{x} > 0 \end{cases}$

x		0		1	
$\frac{x-1}{x}$	$+$	$\|\|$	$-$	0	$+$

---

b) $$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}\Big(\underbrace{-\frac{x}{2}}_{\to\mp\infty} + \underbrace{\ln\underbrace{\left(\frac{x-1}{x}\right)}_{\to 1}}_{\to 0}\Big) = \mp\infty$$

$\Rightarrow$ pas d'AH qd $x\to\pm\infty$

$$f(x) = -\frac{x}{2} + \underbrace{\ln\frac{x-1}{x}}_{=\varphi(x)} \quad \text{avec } \lim_{x\to\pm\infty}\varphi(x) = 0$$

$$\Rightarrow \text{AO}: y = -\frac{x}{2} \quad \text{qd } x\to\pm\infty$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\Big(\underbrace{-\frac{x}{2}}_{\to -1/2} + \underbrace{\ln\underbrace{\left(\frac{x-1}{x}\right)}_{\to 0^+}}_{\to-\infty}\Big) = -\infty \quad \Rightarrow \text{AV}: x=1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\Big(\underbrace{-\frac{x}{2}}_{\to 0} + \ln\underbrace{\left(\frac{x-1}{x}\right)}_{\to+\infty}\Big) = +\infty \quad \Rightarrow \text{AV}: x=0$$

c) $\forall x \in \text{Dom } f$,

$$f'(x) = -\frac{1}{2} + \frac{\frac{x\cdot 1 - 1\cdot(x-1)}{x^2}}{\frac{x-1}{x}} = -\frac{1}{2} + \frac{1}{x(x-1)} = \frac{-x^2+x+2}{2x(x-1)}$$

$f'(x) = 0 \Leftrightarrow -x^2+x+2=0 \qquad \Delta = 9 > 0$

$\Leftrightarrow x = \frac{-1-3}{-2} = 2$ ou $x = \frac{-1+3}{-2} = -1$

$f'(x) > 0 \Leftrightarrow -x^2+x+2 > 0 \qquad \left(2x(x-1) > 0 \text{ vu que } \frac{x-1}{x} > 0 \right)$

$-x^2+x+2 > 0 \Leftrightarrow x \in]-1;2[$

x		-1		2	
$-x^2+x+2$	$-$	0	$+$	0	$-$

d)

x	$-\infty$		-1		0	1		2		$+\infty$
$f'(x)$		$-$	0	$+$	||	||	$+$	0	$-$	
$f(x)$	$+\infty$	$\searrow$	$\frac{1}{2}+\ln 2$ min	$\nearrow$	$+\infty$ ||	|| $-\infty$	$\nearrow$	$-1-\ln 2$ max	$\searrow$	$-\infty$

2)a) $\forall x \in]1,+\infty[$, $f(x) \leq -1-\ln 2 < 0$ (voir tableau)

b) $\mathcal{A} = -\int_2^3 f(x)\,dx = \int_2^3 \left(\frac{x}{2} - \ln\frac{x-1}{x}\right)dx = \left[\frac{1}{4}x^2\right]_2^3 - \int_2^3 \ln\frac{x-1}{x}\,dx$

$\int \ln\frac{x-1}{x}\,dx \qquad$ IPP ; $u(x) = \ln\frac{x-1}{x}$ et $v'(x) = 1$

$\Rightarrow u'(x) = \frac{1}{x(x-1)}$ et $v(x) = x$

(voir calcul de $f'(x)$)

$= x\ln\frac{x-1}{x} - \int \frac{1}{x-1}\,dx$

$= x\ln\frac{x-1}{x} - \ln(x-1) + k$

$\mathcal{A} = \frac{9}{4} - 1 - \left[x\ln\frac{x-1}{x} - \ln(x-1)\right]_2^3$

$= \frac{5}{4} - \left(3\ln\frac{2}{3} - \ln 2 - 2\ln\frac{1}{2} + \underbrace{\ln 1}_{=0}\right)$

$= \frac{5}{4} - (3\ln 2 - 3\ln 3 - \ln 2 + 2\ln 2)$

$= \frac{5}{4} - (4\ln 2 - 3\ln 3) = \left(\frac{5}{4} + \ln\frac{27}{16}\right)\text{ua} \simeq 1{,}77\text{ ua}$

$\boxed{IV}$ $f(x) = \dfrac{5\cdot e^{x}}{e^{2x}+1}$

1) Etude de f

a) $\mathrm{Dom}\, f = \mathbb{R}$ (CE: /)

b) $$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\overbrace{5\cdot e^{x}}^{\to+\infty}}{\underbrace{e^{2x}+1}_{\to+\infty}} \qquad \text{f.i. } \frac{+\infty}{+\infty}$$

$$\overset{(H)}{=} \lim_{x\to+\infty} \frac{5\cdot e^{x}}{2e^{2x}} = \lim_{x\to+\infty} \frac{5}{\underbrace{2e^{x}}_{\to+\infty}} = 0 \Rightarrow AH: y=0 \text{ qd } x\to+\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5\cdot \overbrace{e^{x}}^{\to 0}}{\underbrace{(\underbrace{e^{2x}}_{\to 0}+1)}_{\to 1}} = 0 \Rightarrow AH: y=0 \text{ qd } x\to-\infty$$

c) $\forall x\in\mathbb{R}$,

$$f'(x) = 5\cdot\frac{(e^{2x}+1)\cdot e^{x} - 2\cdot e^{2x}\cdot e^{x}}{(e^{2x}+1)^2} = \frac{5\cdot e^{x}\,(e^{2x}+1-2e^{2x})}{(e^{2x}+1)^2}$$

$$= \frac{5\cdot e^{x}\,(1-e^{2x})}{(e^{2x}+1)^2}$$

$f'(x)=0 \Leftrightarrow 1-e^{2x}=0 \Leftrightarrow e^{2x}=1 \Leftrightarrow x=0$

$f'(x) \gtrless 0 \Leftrightarrow 1-e^{2x} \gtrless 0 \Leftrightarrow e^{2x} \lessgtr 1 \Leftrightarrow x \lessgtr 0$

d) $\forall x\in\mathbb{R}$,

$$f''(x) = 5\cdot\frac{(e^{2x}+1)^{\cancel{2}}\left[e^{x}(1-e^{2x}) + e^{x}\cdot(-2)\cdot e^{2x}\right] - 2\cancel{(e^{2x}+1)}\cdot 2e^{2x}\cdot e^{x}(1-e^{2x})}{(e^{2x}+1)^{\cancel{4}\,3}}$$

$$= 5\cdot\frac{(e^{2x}+1)(e^{x}-e^{3x}-2e^{3x}) - 4e^{3x} + 4e^{5x}}{(e^{2x}+1)^3}$$

$$= 5\cdot\frac{e^{3x} - 3e^{5x} + e^{x} - 3e^{3x} - 4e^{3x} + 4e^{5x}}{(e^{2x}+1)^3}$$

$$= \frac{5\cdot(e^{5x} - 6e^{3x} + e^{x})}{(e^{2x}+1)^3} = \frac{5e^{x}(e^{4x} - 6e^{2x} + 1)}{(e^{2x}+1)^3}$$

$f''(x)=0 \Leftrightarrow e^{4x} - 6e^{2x} + 1 = 0$. Poser $t = e^{2x}$

$\Leftrightarrow t^2-6t+1=0 \Leftrightarrow t=3-2\sqrt{2}$ ou $t=3+2\sqrt{2} \Leftrightarrow e^{2x}=3-2\sqrt{2}$ ou $e^{2x}=3+2\sqrt{2}$

$\Leftrightarrow x=\frac{1}{2}\ln(3-2\sqrt{2}) \simeq -0{,}88$ ou $x=\frac{1}{2}\ln(3+2\sqrt{2}) \simeq 0{,}88$

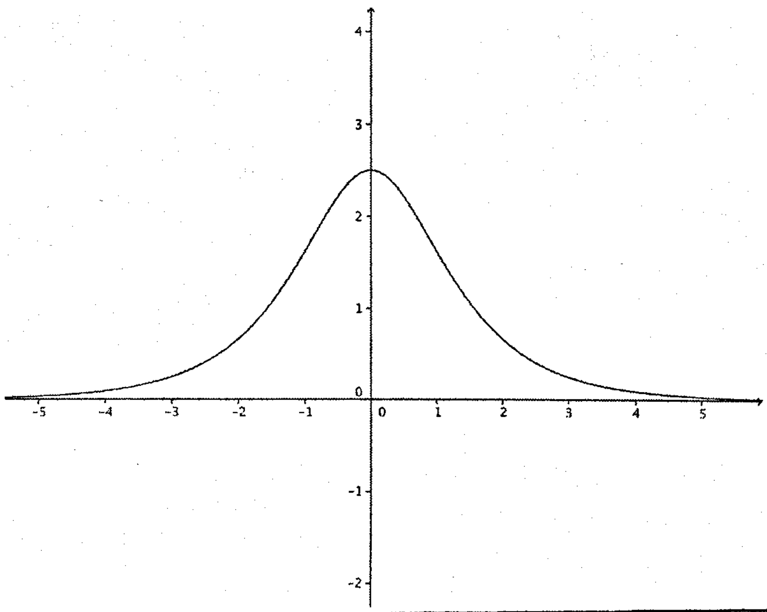
$f''(x)>0 \Leftrightarrow t<3-2\sqrt{2}$ ou $t>3+2\sqrt{2} \Leftrightarrow x<\frac{1}{2}\ln(3-2\sqrt{2})$ ou $x>\frac{1}{2}\ln(3+2\sqrt{2})$

e)

x	$-\infty$		$\frac{1}{2}\ln(3-2\sqrt{2})$		0		$\frac{1}{2}\ln(3+2\sqrt{2})$		$+\infty$
$f''(x)$		$+$	0	$-$		$-$	0	$+$	
$f'(x)$		$+$		$+$	0	$-$		$-$	
$f(x)$	0	$\nearrow$		$\nearrow$	$\frac{5}{2}$	$\searrow$		$\searrow$	0
$\mathcal{C}_f$	AH: $y=0$	$\smile$	PI1	$\frown$	max	$\frown$	PI2	$\smile$	AH: $y=0$

$f\left(\frac{1}{2}\ln(3-2\sqrt{2})\right) \simeq 1{,}77$; $f\left(\frac{1}{2}\ln(3+2\sqrt{2})\right) \simeq 1{,}77$

f)



2) $T: y=f(\ln 2)+f'(\ln 2)\cdot(x-\ln 2)$ $\quad f(\ln 2)=\frac{5\cdot 2}{5}=2$; $f'(\ln 2)=\frac{5\cdot 2\cdot(-3)}{5^2}=-\frac{6}{5}$

$\Leftrightarrow y=2-\frac{6}{5}(x-\ln 2) \Leftrightarrow y=-\frac{6}{5}x+\left(2+\frac{6}{5}\cdot\ln 2\right)$

3) a) $\mathcal{A}=\int_0^{\ln\sqrt{3}} f(x)\,dx=\int_0^{\ln\sqrt{3}} \frac{5e^x}{e^{2x}+1}\,dx=5\int_0^{\ln\sqrt{3}} \frac{e^x}{1+(e^x)^2}\,dx=5\left[\operatorname{Arctan}(e^x)\right]_0^{\ln\sqrt{3}}$

$=5\left(\operatorname{Arctan}\sqrt{3}-\operatorname{Arctan}1\right)=5\cdot\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{5\pi}{12}$ ua

b) $V=\pi\int_0^{\ln\sqrt{3}} [f(x)]^2\,dx=\pi\int_0^{\ln\sqrt{3}} \frac{25e^{2x}}{(e^{2x}+1)^2}\,dx=\frac{25\pi}{2}\int_0^{\ln\sqrt{3}} \underbrace{2e^{2x}}_{u'(x)}(\underbrace{e^{2x}+1}_{u(x)})^{-2}\,dx$

$=-\frac{25\pi}{2}\left[\frac{1}{e^{2x}+1}\right]_0^{\ln\sqrt{3}}=-\frac{25\pi}{2}\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{25\pi}{8}$ uv