Corrigé

I. 1) 2) voir livre p. 26, 28, 67

3) $f(x) = \text{Arc}\cos \dfrac{x+2}{x-2}$

a) C.E. ① $x-2 \neq 0 \iff x \neq 2$

② $-1 < \dfrac{x+2}{x-2} < 1 \iff \dfrac{x+2}{x-2} + 1 > 0$ et $\dfrac{x+2}{x-2} - 1 < 0$

$$\iff \begin{cases} \dfrac{2x}{x-2} > 0 \\ \dfrac{4}{x-2} < 0 \end{cases} \iff \begin{cases} x \in]\leftarrow ; 0[\cup]2 ; \rightarrow \\ x \in]\leftarrow ; 2[\end{cases}$$

$\text{dom } f' =]\leftarrow ; 0[$

b)

$$f'(x) = - \frac{\dfrac{x-2-(x+2)}{(x-2)^2}}{\sqrt{1-\left(\dfrac{x+2}{x-2}\right)^2}}$$

$$f'(x) = \frac{4}{(x-2)^2 \sqrt{1-\left(\dfrac{x+2}{x-2}\right)^2}}$$

II. 1) $e^{2x-1} + e^x = e^{-1} + 1$ $\quad D = \mathbb{R}$

$\iff e^{2x} \cdot e^{-1} + e^x = e^{-1} + 1 \quad | \cdot e$

$\iff e^{2x} + e \cdot e^x = 1 + e$

$\iff e^{2x} + e \cdot e^x - (1+e) = 0$

[posons $y = e^x > 0$

$\iff y^2 + ey - (1+e) = 0$

$\Delta = e^2 + 4(1+e) = e^2 + 4e + 4 = (e+2)^2 > 0$

$y = \dfrac{-e+e+2}{2} = 1$

ou

$y = \dfrac{-e-e-2}{2} = \dfrac{-2e-2}{2} = -(e+1) < 0$ à écarter

Or, $y = e^x \iff 1 = e^x \iff 0 = x$

$S = \{0\}$

2) $(0{,}4)^{x^2+7x} > (6{,}25)^{3-x}$ $\quad D = \mathbb{R}$

$\iff \left(\dfrac{2}{5}\right)^{x^2+7x} > \left(\dfrac{25}{4}\right)^{3-x}$

$\iff \left(\dfrac{2}{5}\right)^{x^2+7x} > \left[\left(\dfrac{2}{5}\right)^{-2}\right]^{3-x}$

$\iff \left(\dfrac{2}{5}\right)^{x^2+7x} > \left(\dfrac{2}{5}\right)^{-6+2x}$ $\quad \boxed{a = \dfrac{2}{5} < 1}$

$\iff x^2 + 7x < -6 + 2x$

$\iff x^2 + 5x + 6 < 0$

posons $x^2 + 5x + 6 = 0$

$\Delta = 1$

$x = \dfrac{-5+1}{2} = -2$

ou

$x = \dfrac{-5-1}{2} = -3$

$S =]-3 ; -2[$

II. 3) $\ln\sqrt{2x+3} \le \ln\sqrt{e} - \frac{1}{2}\ln(1-2x)$

C.E. ① $2x+3 \geq 0 \iff x \geq -\frac{3}{2}$

② $\sqrt{2x+3} > 0 \iff x > -\frac{3}{2}$

③ $1-2x > 0 \iff x < \frac{1}{2}$

$D = \left]-\frac{3}{2} ; \frac{1}{2}\right[$

$\iff \frac{1}{2}\ln(2x+3) \le \frac{1}{2}\ln e - \frac{1}{2}\ln(1-2x) \quad |\cdot 2$

$\iff \ln(2x+3) \le \ln e - \ln(1-2x)$

$\iff \ln(2x+3) + \ln(1-2x) \le \ln e$

$\iff \ln[(2x+3)(1-2x)] \le \ln e \qquad \boxed{a = e > 1}$

$\iff -4x^2-4x+3 \le e$

$\iff -4x^2-4x+3-e \le 0 \quad |\cdot(-1)$

$\iff \boxed{4x^2+4x-(3-e) \geq 0}$

posons $4x^2+4x-(3-e) = 0$

$\Delta = 16 + 16(3-e) = 16\underbrace{(4-e)}_{>0} > 0$

$x = \frac{-4+4\sqrt{4-e}}{8} = \frac{-1+\sqrt{4-e}}{2} \simeq 0{,}066$

ou

$x = \frac{-1-\sqrt{4-e}}{2} \simeq -1{,}066$

$S = \left(\leftarrow ; \frac{-1-\sqrt{4-e}}{2}\right] \cup \left[\frac{-1+\sqrt{4-e}}{2} ; \rightarrow\right) \cap D$

$S = \left]-\frac{3}{2} ; \frac{-1-\sqrt{4-e}}{2}\right] \cup \left[\frac{-1+\sqrt{4-e}}{2} ; \frac{1}{2}\right[$

III. $f : x \to (x^2+x+1)\underbrace{e^{-x}}_{>0,\ \forall x \in \mathbb{R}}$

1) a) $\mathrm{dom} f = \mathbb{R} = \mathrm{dom} f'$

b) $\lim\limits_{x\to+\infty} f(x) = \text{"}+\infty \cdot 0\text{"}$ f.i.

$= \lim\limits_{x\to+\infty} \frac{x^2+x+1}{e^x} = \text{"}\frac{+\infty}{+\infty}\text{"}$

$\overset{(H)}{=} \lim\limits_{x\to+\infty} \frac{2x+1}{e^x}$

$\overset{(H)}{=} \lim\limits_{x\to+\infty} \frac{2}{e^x}$ ($e^x \to +\infty$)

$\lim\limits_{x\to+\infty} f(x) = 0 \qquad$ A.H. $\boxed{y=0}$ $(x \to +\infty)$

$\lim\limits_{x\to-\infty} f(x) = \text{"}+\infty \cdot +\infty\text{"} = +\infty \qquad$ ~~A.H.~~ $(x \to -\infty)$

<u>recherche d'une A.O. $(x \to -\infty)$</u>

$\lim\limits_{x\to-\infty} \frac{f(x)}{x} = \lim\limits_{x\to-\infty}\left(\underbrace{\frac{x^2+x+1}{x}}_{-\infty} \cdot \underbrace{e^{-x}}_{+\infty}\right) = -\infty \qquad$ ~~A.O.~~

c) $f'(x) = (2x+1)e^{-x} + (x^2+x+1)(-e^{-x})$

$= e^{-x}(-x^2+x)$

$f'(x) = \underbrace{e^{-x}}_{>0} x \cdot (-x+1)$

$f'(x) = 0 \iff x = 0$ ou $x = 1$

x	$-\infty$		0		1		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	$+\infty$	↘	1	↗	$\frac{3}{e}$	↘	0

1) d) $f'(x) = e^{-x}(-x^2+x)$

$f''(x) = -e^{-x}(-x^2+x) + e^{-x}(-2x+1)$

$\Leftrightarrow f''(x) = e^{-x}(x^2-3x+1)$

$f''(x) = 0 \Leftrightarrow x^2-3x+1=0$

$\Delta = 5$

$x = \frac{3+\sqrt5}{2} \simeq 2{,}62$

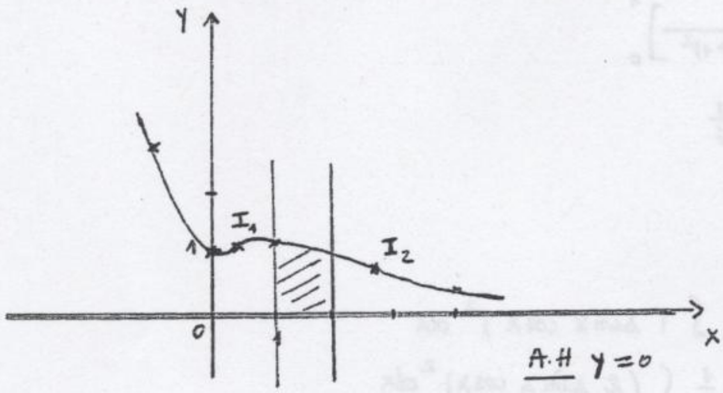
ou

$x = \frac{3-\sqrt5}{2} \simeq 0{,}38$

G_f admet deux points d'inflexions. $I_1(0{,}38 ; 1{,}04)$ et $I_2(2{,}62 ; 0{,}76)$

e)

x	-1	0	$0{,}38$	1	$2{,}62$	4
$f(x)$	e	1	$1{,}04$	$1{,}1$	$0{,}76$	$0{,}38$



2) $A = \int_1^2 f(x)\,dx$

$= \int_1^2 [\underbrace{(x^2+x+1)}_{f} \underbrace{e^{-x}}_{g'}]\,dx$

[posons $f(x) = x^2+x+1$ $f'(x) = 2x+1$
$g'(x) = e^{-x}$ $g(x) = -e^{-x}$

$= [(x^2+x+1)e^{-x}]_1^2 + \int_1^2 \underbrace{(2x+1)}_{f} \underbrace{e^{-x}}_{g'}\,dx$

[posons $f(x) = 2x+1$ $f'(x) = 2$
$g'(x) = e^{-x}$ $g(x) = -e^{-x}$

$= [-(x^2+x+1)e^{-x} - (2x+1)e^{-x}]_1^2 + 2\int_1^2 e^{-x}\,dx$

$= [-(x^2+x+1)e^{-x} - (2x+1)e^{-x} - 2e^{-x}]_1^2$

$= [-e^{-x}(x^2+3x+4)]_1^2$

$= -\frac{14}{e^2} + \frac{8}{e}$

$A \simeq 1{,}05$ u.a.

IV. 1) $\displaystyle\int \frac{3x-2}{\sqrt{5-x^2}}\,dx = 3\int \frac{x}{\sqrt{5-x^2}}dx - 2\int \frac{1}{\sqrt{5-x^2}}\,dx$

$$= 3\int \frac{2x}{2\sqrt{5-x^2}}\,dx - 2\int \frac{1}{\sqrt{5}\sqrt{1-\left(\frac{x}{\sqrt{5}}\right)^2}}\,dx$$

$$= 3\sqrt{5-x^2} - 2\,\mathrm{Arc}\sin\left(\frac{x}{\sqrt{5}}\right) + k,\ k\in\mathbb{R}$$

2) $\displaystyle\int_0^1 \underset{f}{(x+3)}\underset{g'}{(x+1)^{-4}}\,dx$

$\left[\text{posons } \begin{array}{ll} f(x) = x+3 & f'(x) = 1 \\ g'(x) = (x+1)^{-4} & g(x) = -\frac{1}{3}(x+1)^{-3} \end{array}\right.$

$$= -\frac{x+3}{3(x+1)^3}\Big]_0^1 + \frac{1}{3}\int_0^1 (x+1)^{-3}\,dx$$

$$= \left[-\frac{x+3}{3(x+1)^3} - \frac{1}{6(x+1)^2}\right]_0^1$$

$$= -\frac{4}{24} - \frac{1}{24} + 1 + \frac{1}{6}$$

$$= \frac{23}{24}$$

3) $\displaystyle\int \sin^2 x\cos^2 x\,dx = \int (\sin x\cos x)^2\,dx$

$$= \frac{1}{4}\int (2\sin x\cos x)^2 dx$$

$$= \frac{1}{4}\int \sin^2 2x\,dx$$

$$= \frac{1}{4}\cdot\frac{1}{2}\int (1-\cos 4x)\,dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + k,\ k\in\mathbb{R}$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + k$$