I ⓐ $6iz^2 - 5(1+2i)z + 17 = 0$

$$\Delta = 25(1+2i)^2 - 4\cdot 6\cdot 17 i$$
$$= 25(1+4i-4) - 408i$$
$$= -75 - 308i$$

rac. carrées de Δ : $\pm\delta = \pm(a+bi)$

$$\begin{cases} a^2+b^2 = \sqrt{75^2+308^2} = 317 \\ a^2-b^2 = -75 \\ 2ab = -308 \text{ [signes diff.]} \end{cases} \quad \begin{matrix} 2a^2 = 242 & 2b^2 = 392 \\ a^2 = 121 & b^2 = 196 \\ a = \pm 11 & b = \pm 14 \end{matrix}$$

$$\pm\delta = \pm(11-14i)$$

solutions : $z_1 = \dfrac{5+10i+11-14i}{12i} = \dfrac{16-4i}{12i} = \dfrac{4-i}{3i} = \dfrac{-1-4i}{3}$

$$z_2 = \frac{5+10i-11+14i}{12i} = \frac{-6+24i}{12i} = \frac{-1+4i}{2i} = \frac{4+i}{2}$$

$$S = \left\{ \frac{-1-4i}{3} ; \frac{4+i}{2} \right\}$$

ⓑ 1) $z_1 = -\sqrt{2}+\sqrt{2}\,i$; $z_1^2 = 2-4i-2 = -4i$; $z_1^4 = -16$; $z_1^5 = 16\sqrt{2}(1-i)$

$z_2 = 3\sqrt{2} - \sqrt{6}\,i$; $z_2^2 = 18 - 12\sqrt{3}\,i - 6 = 12(1-\sqrt{3}\,i)$

$$\frac{z_1^5}{z_2^2} = \frac{16\sqrt{2}(1-i)}{12(1-\sqrt{3}i)} = \frac{4\sqrt{2}(1-i)(1+\sqrt{3}i)}{3\cdot(1+3)} = \frac{\sqrt{2}}{3}\cdot(1+\sqrt{3}i - i + \sqrt{3})$$
$$= \frac{(\sqrt{6}+\sqrt{2}) + i(\sqrt{6}-\sqrt{2})}{3}$$

2) $z_1 = 2 \operatorname{cis}\frac{3\pi}{4}$; $z_2 = 2\sqrt{6}\operatorname{cis}(-\frac{\pi}{6})$

$$\frac{z_1^5}{z_2^2} = \frac{2^5 \operatorname{cis}\frac{15\pi}{4}}{24\operatorname{cis}(-\frac{\pi}{3})} = \frac{4}{3}\operatorname{cis}\left(\frac{15\pi}{4}+\frac{\pi}{3}\right) = \frac{4}{3}\operatorname{cis}\frac{49\pi}{12} = \frac{4}{3}\operatorname{cis}\frac{\pi}{12}$$

3) $$\cos\frac{\pi}{12} = \frac{(\sqrt{6}+\sqrt{2})\cdot 3}{3\cdot 4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin\frac{\pi}{12} = \frac{(\sqrt{6}-\sqrt{2})\cdot 3}{3\cdot 4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

II (a) $\begin{cases} x+2y+az=2 \\ -x+ay+2z=a \\ 2x+y-az=a \end{cases}$ $\Delta = \begin{vmatrix} 1 & 2 & a \\ -1 & a & 2 \\ 2 & 1 & -a \end{vmatrix} = -a^2+8-a-2a^2-2-2a$

$= -3a^2-3a+6$

$= -3\cdot(a-1)(a+2)$

$\boxed{a=1}$ $\begin{cases} x+2y+z=2 & (1) \\ -x+y+2z=1 & (2) \\ 2x+y-z=1 & (3) \end{cases}$

x de (1) : $x=2-2y-z$ ds (2)(3) : $\begin{cases} -2+2y+z+y+2z=1 \\ 4-4y-2z+y-z=1 \end{cases}$

$\begin{cases} 3y+3z=3 \\ -3y-3z=-3 \end{cases}$

$y+z=1$

$y=1-z$ d'où $x=2-2(1-z)-z$

$x=z$

$S=\{(z;1-z;z) \mid z\in\mathbb{R}\}$ $\infty^{té}$ de solutions

$\boxed{a=-2}$ $\begin{cases} x+2y-2z=2 \\ -x-2y+2z=-2 \\ 2x+y+2z=-2 \end{cases}$ $\begin{cases} x+2y-2z=2 & (1) \\ 2x+y+2z=-2 & (2) \end{cases}$

x de (1) : $x=2-2y+2z$ (*)

dans (2) $4-4y+4z+y+2z=-2$

$-3y+6z=-6$

$y=2z+2$

ds (*) : $x=2-4z-4+2z$

$x=-2z-2$

$S=\{(-2z-2;2z+2;z) \mid z\in\mathbb{R}\}$ $\infty^{té}$ de solutions

$\boxed{a\in\mathbb{R}-\{1;-2\}}$ solution unique

$\Delta_x = \begin{vmatrix} 2 & 2 & a \\ a & a & 2 \\ a & 1 & -a \end{vmatrix} = -2a^2+4a+a^2-a^3-4+2a^2$

$= -a^3+a^2+4a-4$

$= -a^2(a-1)+4(a-1)$

$=(a-1)(2-a)(2+a)$

$\Delta_y = \begin{vmatrix} 1 & 2 & a \\ -1 & a & 2 \\ 2 & a & -a \end{vmatrix} = -a^2+8-a^2-2a^2-2a-2a$

$=-4a^2-4a+8$

$=-4(a-1)(a+2)$

$\Delta_z = \begin{vmatrix} 1 & 2 & 2 \\ -1 & a & a \\ 2 & 1 & a \end{vmatrix} = a^2+4a-2-4a-a+2a$

$=a^2+a-2$

$=(a-1)(a+2)$

$x=\dfrac{(a-1)(2-a)(2+a)}{-3(a-1)(a+2)}$ $y=\dfrac{-4(a-1)(a+2)}{-3(a-1)(a+2)}$ $z=\dfrac{(a-1)(a+2)}{-3(a-1)(a+2)}$

$x=\dfrac{a-2}{3}$ $y=\dfrac{4}{3}$ $z=-\dfrac{1}{3}$

$S=\left\{\left(\dfrac{a-2}{3};\dfrac{4}{3};-\dfrac{1}{3}\right)\right\}$

(b) $A \cdot B = \begin{pmatrix} 2 & -1 & -1 \\ 1 & -2 & -1 \\ 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ 2 & -1 & 0 \\ 1 & 0 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 3 & 6 \\ -5 & 3 & 4 \\ 11 & -3 & 2 \end{pmatrix}$

$B - AB = \begin{pmatrix} 3 & -2 & -4 \\ 7 & -4 & -4 \\ -10 & 3 & -4 \end{pmatrix}$ $\det(B-AB) = 24$ donc $B-AB$ inversible.

$(B-AB)^{-1} = \frac{1}{24}\begin{pmatrix} 28 & -20 & -8 \\ 68 & -52 & -16 \\ -19 & 11 & 2 \end{pmatrix} = \begin{pmatrix} 7/6 & -5/6 & -1/3 \\ 17/6 & -13/6 & -2/3 \\ -19/24 & 11/24 & 1/12 \end{pmatrix}$

III (a) 1) $C_4^1 \cdot A_8^4 = 4 \cdot 1680 = 6\,720$ 1 2 3 4 5 6 7 8 9

2) $C_6^2 \cdot 3! \cdot 3! = 15 \cdot 6 \cdot 6 = 540$ $\cdot\cdot\ \boxed{756}$

(b) 1) $C_4^2 \cdot C_4^2 \cdot C_{44}^2 = 6 \cdot 6 \cdot 946 = 34\,056$ cas possibles: $C_{52}^6 = 20\,358\,520$

$p_1 = 0{,}0017$

2) 5 coeurs: $C_{13}^5 \cdot C_{39}^1 = 1287 \cdot 39 = 50\,193$
6 coeurs: $C_{13}^6 \cdot C_{39}^0 = 1716 \cdot 1 = 1716$ } $51\,909$

$p_2 = \frac{4 \cdot 51\,909}{C_{52}^6} = \frac{207\,636}{C_{52}^6} = 0{,}0102$

(c) $\begin{pmatrix} 6R \\ 8N \end{pmatrix} \to 4$

4R → 16 EUR
3R 1N → 9 EUR
2R 2N → 2 EUR
1R 3N → −5 EUR
4N → −12 EUR

Calcul des proba: cas poss. $C_{14}^4 = 1001$

$x_1 = 16$ cas fav. $C_6^4 \cdot C_8^0 = 15 \cdot 1 = 15$ $p_1 = 0{,}0150$

$x_2 = 9$ cas fav. $C_6^3 \cdot C_8^1 = 20 \cdot 8 = 160$ $p_2 = 0{,}1598$

$x_3 = 2$ cas fav. $C_6^2 \cdot C_8^2 = 15 \cdot 28 = 420$ $p_3 = 0{,}4196$

$x_4 = -5$ cas fav. $C_6^1 \cdot C_8^3 = 6 \cdot 56 = 336$ $p_4 = 0{,}3357$

$x_5 = -12$ cas fav. $C_6^0 \cdot C_8^4 = 1 \cdot 70 = 70$ $p_5 = 0{,}0699$

loi de probabilités de X :

x_i	16	9	2	−5	−12
$p(X=x_i)$	0,0150	0,1598	0,4196	0,3357	0,0699

Espérance de X : $E(X) = 0$

Variance de X : $V(X) = 25{,}5013$

Ecart-type : $\sigma(X) = 5{,}05$