EXAMEN DE FIN D'ÉTUDES SECONDAIRES CLASSIQUES

Sessions 2023 – CORRIGÉ-BARÈME ÉCRIT

Date :	15.09.23	Durée :	08:15 - 12:15
Discipline :	Mathématiques II - Informatique - Mathématiques 2	Section(s) :	CB / CB-4LANG

Question 1 (6 + 4 + 4 = 14 points)

1) $\forall x \in \mathbb{R}$: $(m-7)4^{-x}-(m+3)4^{x}=m \quad |\cdot 4^{x}>0$

$\Leftrightarrow (m-7)-(m+3)4^{2x}=m4^{x}$

$\Leftrightarrow -(m+3)4^{2x}-m4^{x}+(m-7)=0 \quad (E_x)$

[1] En posant $y=4^{x}>0$, (E_x) devient $-(m+3)y^2-my+(m-7)=0 \quad (E_y)$

[1] Si $m=-3$, alors $(E_y) \Leftrightarrow 3y-10=0 \Leftrightarrow y=\frac{10}{3}(>0)$ donc (E_x) admet une seule solution.

[1] Si $m \neq -3$, alors $\Delta = m^2+4(m+3)(m-7)=5m^2-16m-84$

$=(m-6)(5m+14)=5(m-6)\left(m+\frac{14}{5}\right)$,

$P=-\dfrac{m-7}{m+3}$ et $S=-\dfrac{m}{m+3}$.

m	$-\infty$		-3		$-\frac{14}{5}$		0		6		7		$+\infty$
Δ		+	‖	+	0	−	−	−	0	+	+	+	
P		−	‖	+	+	+	+	+	+	+	0	−	
S		−	‖	+	+	+	0	−	−	−	−	−	
nbre sol (E_y)		2	1	2	1	0	0	0	1	2	2	2	
[3] nbre sol (E_x)		1	1	2	1	0	0	0	0	0	0	1	

En résumé :

(E_x) n'admet pas de solution $\Leftrightarrow m \in \left]-\frac{14}{5};7\right]$

(E_x) admet une seule solution $\Leftrightarrow m \in \left]-\infty;-3\right] \cup \left\{-\frac{14}{5}\right\} \cup \left]7;+\infty\right[$

(E_x) admet deux solutions distinctes $\Leftrightarrow m \in \left]-3;-\frac{14}{5}\right[$

2) a) $\log_3|x-3| = \log_{\frac{1}{3}}(x+1) + \log_{\sqrt{3}}(2-x) \quad (E)$

CE : $x \neq 3 \wedge x > -1 \wedge x < 2 \Leftrightarrow -1 < x < 2$

[1] Dom $=]-1; 2[$

$(E) \Leftrightarrow \log_3|x-3| = \dfrac{\log_3(x+1)}{\log_3 \frac{1}{3}} + \dfrac{\log_3(2-x)}{\log_3 \sqrt{3}}$

$\Leftrightarrow \log_3(3-x) = -\log_3(x+1) + 2\log_3(2-x)$

$\Leftrightarrow \log_3[(3-x)(x+1)] = \log_3(2-x)^2$

$\Leftrightarrow (3-x)(x+1) = (2-x)^2$

$\Leftrightarrow 3 + 2x - x^2 = 4 - 4x + x^2$

$\Leftrightarrow 2x^2 - 6x + 1 = 0 \; (\Delta = 36 - 8 = 28)$

[2,5] $\Leftrightarrow x = \frac{6\pm2\sqrt{7}}{4} = \frac{3\pm\sqrt{7}}{2} \left(\simeq < \begin{matrix} 2{,}82 \\ 0{,}18 \end{matrix}\right)$

[0,5] $S = \left\{\frac{3-\sqrt{7}}{2}; \frac{3+\sqrt{7}}{2}\right\} \cap \text{Dom} = \left\{\frac{3-\sqrt{7}}{2}\right\}$

2) b) $6^{x+\frac{3}{2}} - 5^{2x} \geq 6^{x+\frac{1}{2}} + 5^{2x+1}$ Dom $= \mathbb{R}$

$\Leftrightarrow 6^{x+\frac{3}{2}} - 6^{x+\frac{1}{2}} \geq 5^{2x} + 5^{2x+1}$

$\Leftrightarrow (6-1)6^{x+\frac{1}{2}} \geq (1+5)5^{2x}$

$\Leftrightarrow 5 \cdot 6^{x+\frac{1}{2}} \geq 6 \cdot 5^{2x}$

$\Leftrightarrow 6^{x-\frac{1}{2}} \geq 5^{2x-1}$

$\Leftrightarrow e^{\left(x-\frac{1}{2}\right)\ln 6} \geq e^{(2x-1)\ln 5} \quad |\ln(\;\;) \nearrow$

$\Leftrightarrow \left(x-\frac{1}{2}\right)\ln 6 \geq (2x-1)\ln 5$

$\Leftrightarrow \left(x-\frac{1}{2}\right)\ln 6 \geq 2\left(x-\frac{1}{2}\right)\ln 5$

$\Leftrightarrow \left(x-\frac{1}{2}\right)\left(\underbrace{\ln 6 - 2\ln 5}_{<0}\right) \geq 0$

$\Leftrightarrow x - \frac{1}{2} \leq 0$

$\Leftrightarrow x \leq \frac{1}{2}$

[4] $S = \left]-\infty; \frac{1}{2}\right] \cap \text{Dom} = \left]-\infty; \frac{1}{2}\right]$

Question 2 (2 + 4 = 6 points)

1)

0,5 $I \subset \mathbb{R} \setminus \{k\pi | k \in \mathbb{Z}\}$

$$\int \frac{\cos^3 x}{\sin x}\,dx$$

$$= \int \frac{\cos^2 x}{\sin x} \cos x\,dx \qquad \left| \begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array} \right.$$

$$= \int \frac{1 - t^2}{t}\,dt$$

$$= \int \left(\frac{1}{t} - t\right) dt$$

$$= \ln|t| - \frac{1}{2}t^2 + c$$

1,5 $= \ln|\sin x| - \frac{1}{2}\sin^2 x + c \ (c \in \mathbb{R})$

2)

$$\forall x \in \mathbb{R}_0^+ \colon A(x) = \int \cos(\ln x)\,dx \qquad \left| IPP\colon \begin{array}{ll} u_1(x) = \cos(\ln x) & u_1'(x) = -\sin(\ln x) \cdot \frac{1}{x} \\ v'(x) = 1 & v(x) = x \end{array} \right.$$

$$= x\cos(\ln x) + \int \sin(\ln x)\,dx \qquad \left| IPP\colon \begin{array}{ll} u_2(x) = \sin(\ln x) & u_2{}'(x) = \cos(\ln x) \cdot \frac{1}{x} \\ v'(x) = 1 & v(x) = x \end{array} \right.$$

$$= x\cos(\ln x) + x\sin(\ln x) - \underbrace{\int \cos(\ln x)\,dx}_{A(x)}$$

2,5 $\forall x \in \mathbb{R}_0^+ \colon A(x) = \frac{1}{2}x[\cos(\ln x) + \sin(\ln x)] + c \ (c \in \mathbb{R})$

$$\int_1^{e^{-\frac{3\pi}{4}}} \cos(\ln x)\,dx = A\left(e^{-\frac{3\pi}{4}}\right) - A(1)$$

$$= \frac{1}{2}e^{-\frac{3\pi}{4}}\left[\cos\left(-\frac{3\pi}{4}\right) + \sin\left(-\frac{3\pi}{4}\right)\right] - \frac{1}{2}(\cos 0 + \sin 0)$$

$$= \frac{1}{2}e^{-\frac{3\pi}{4}}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - \frac{1}{2}(1 + 0)$$

1,5 $= -\frac{\sqrt{2}}{2}e^{-\frac{3\pi}{4}} - \frac{1}{2} (\simeq -0{,}57)$

Question 3 (3 + 3 + 5 = 11 points)

1)

Dom $f_m = \mathbb{R}\backslash\{-1\}$

1 $\lim\limits_{x\to\pm\infty} f_m(x) = \lim\limits_{x\to\pm\infty} e^{\overbrace{\frac{mx-2}{x+1}}^{\to m}} = e^m$ AH $\equiv y = e^m$

$\lim\limits_{x\to-1}(mx-2) = -m-2 \begin{cases} >0 \text{ si } m<-2 \\ <0 \text{ si } m>-2 \end{cases}$

1 1^er^ cas : $\boxed{m<-2}$

$\lim\limits_{x\to(-1)^-} f_m(x) = \lim\limits_{x\to(-1)^-} e^{\overbrace{\frac{\overbrace{mx-2}^{>0}}{\underbrace{x+1}_{\to 0^-}}}^{\to-\infty}} = 0$ point creux $(-1;0)$

$\lim\limits_{x\to(-1)^+} f_m(x) = \lim\limits_{x\to(-1)^+} e^{\overbrace{\frac{\overbrace{mx-2}^{>0}}{\underbrace{x+1}_{\to 0^+}}}^{\to+\infty}} = +\infty$ AVD $\equiv x=-1$

1 2^e^ cas : $\boxed{m>-2}$

$\lim\limits_{x\to(-1)^-} f_m(x) = \lim\limits_{x\to(-1)^-} e^{\overbrace{\frac{\overbrace{mx-2}^{<0}}{\underbrace{x+1}_{\to 0^-}}}^{\to+\infty}} = +\infty$ AVG $\equiv x=-1$

$\lim\limits_{x\to(-1)^+} f_m(x) = \lim\limits_{x\to(-1)^+} e^{\overbrace{\frac{\overbrace{mx-2}^{<0}}{\underbrace{x+1}_{\to 0^+}}}^{\to-\infty}} = 0$ point creux $(-1;0)$

2)

1 $\forall x \in \mathbb{R}\backslash\{-1\}: f_m{}'(x) = e^{\frac{mx-2}{x+1}} \cdot \frac{m(x+1)-(mx-2)}{(x+1)^2} = \frac{f_m(x)}{\underbrace{(x+1)^2}_{>0}} \cdot (m+2) \begin{cases} <0 \text{ si } m<-2 \\ >0 \text{ si } m>-2 \end{cases}$

1 1^er^ cas : $\boxed{m<-2}$

x	$-\infty$		-1		$+\infty$
$f_m{}'(x)$		$-$	$\|\|$	$-$	
$f_m(x)$	e^m	↘	$0\|\|+\infty$	↘	e^m

1 2^e^ cas : $\boxed{m>-2}$

x	$-\infty$		-1		$+\infty$
$f_m{}'(x)$		$+$	$\|\|$	$+$	
$f_m(x)$	e^m	↗	$+\infty\|\|0$	↗	e^m

3)

[1,5] $\forall x \in \mathbb{R}\backslash\{-1\}: f_m''(x) = (m+2)\cdot\dfrac{f_m'(x)\cdot(x+1)^2 - f_m(x)\cdot 2(x+1)}{(x+1)^4}$

$$= (m+2)\cdot\frac{f_m(x)\cdot(m+2) - f_m(x)\cdot 2(x+1)}{(x+1)^4}$$

$$= \underbrace{(m+2)}_{\neq 0}\cdot\underbrace{\frac{f_m(x)}{(x+1)^4}}_{>0}\cdot(-2x+m)$$

$f_m''(x) = 0 \Leftrightarrow x = \frac{m}{2}\begin{cases}< -1 \text{ si } m < -2\\ > -1 \text{ si } m > -2\end{cases}$

[1,5] 1[er] cas : $\boxed{m < -2}$

x	$-\infty$		$\frac{m}{2}$		-1		$+\infty$
$f_m''(x)$		$-$	0	$+$	$\|$	$+$	
C_m		$\cap$	PI	$\cup$	$\|$	$\cup$	

[1,5] 2[e] cas : $\boxed{m > -2}$

x	$-\infty$		-1		$\frac{m}{2}$		$+\infty$
$f_m''(x)$		$+$	$\|$	$+$	0	$-$	
C_m		$\cup$	$\|$	$\cup$	PI	$\cap$	

$f_m\left(\frac{m}{2}\right) = e^{\frac{m\frac{m}{2}-2}{\frac{m}{2}+1}} = e^{\frac{m^2-4}{m+2}} = e^{m-2}$

[0,5] Dans les deux cas, C_m admet exactement un seul point d'inflexion de coordonnées $\left(\frac{m}{2}; e^{m-2}\right)$.

Question 4 (2 + 4 + 7 + 4 + 2 = 19 points)

Partie 1 :

$\forall x \in \text{Dom } g = \mathbb{R} = \text{Dom}_d g$: $g'(x) = 1 - e^x$

$g'(x) = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$

$g'(x) > 0 \Leftrightarrow e^x < 1 \Leftrightarrow x < 0$

x	$-\infty$		0		$+\infty$
$g'(x)$		+	0	−	
$g(x)$		↗	0	↘	

[2] On en déduit que $g(0) = 0$ et $\forall x \in \mathbb{R}_0$: $g(x) < 0$.

Partie 2 :

1)

Dom $f = \mathbb{R}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\overbrace{1+(x-1)e^x}^{\to 0}}{\underbrace{1-e^x}_{\to 0}} \overset{\boxed{H}}{=} \lim_{x\to 0^-} \frac{e^x + (x-1)e^x}{-e^x} = \lim_{x\to 0^-} \frac{xe^x}{-e^x} = \lim_{x\to 0^-}(-x) = 0 = f(0)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \underbrace{-x}_{\to 0^-} \underbrace{(1-\ln x)^2}_{\to +\infty} = \lim_{x\to 0^+} \frac{\overbrace{(1-\ln x)^2}^{\to +\infty}}{\underbrace{-\frac{1}{x}}_{\to -\infty}} \overset{\boxed{H}}{=} \lim_{x\to 0^+} \frac{2(1-\ln x)\left(-\frac{1}{x}\right)}{\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{\overbrace{-2(1-\ln x)}^{\to -\infty}}{\underbrace{\frac{1}{x}}_{\to +\infty}} \overset{\boxed{H}}{=} \lim_{x\to 0^+} \frac{2\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-2x) = 0 = f(0)$$

[1,5] Comme $\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$, f est continue en 0.

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{\overbrace{1+(x-1)e^x}^{\to 0}}{\underbrace{x(1-e^x)}_{\to 0}} \overset{\boxed{H}}{=} \lim_{x\to 0^-} \frac{xe^x}{1-e^x-xe^x} = \lim_{x\to 0^-} \frac{\overbrace{x}^{\to 0}}{\underbrace{e^{-x}-1-x}_{\to 0}}$$

$$\overset{\boxed{H}}{=} \lim_{x\to 0^-} \frac{1}{-e^{-x}-1} = -\frac{1}{2} = f'_g(0)$$

$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} -(1-\ln x)^2 = -\infty \notin \mathbb{R}$, donc f n'est pas dérivable à droite de 0.

[2,5] IG: demi-tangente de pente $-\frac{1}{2}$ à gauche de $(0;0)$ et

demi-tangente verticale vers le bas à droite de $(0;0)$

2)

[1] $$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1+\overbrace{(x-1)e^x}^{\to 0\ (\text{CAP})}}{\underbrace{1-e^x}_{\to 1}} = 1 \quad \text{AHG} \equiv y = 1$$

$$\text{CAP: } \lim_{x\to-\infty} \overbrace{(x-1)}^{\to-\infty}\overbrace{e^x}^{\to 0} = \lim_{x\to-\infty} \frac{\overbrace{x-1}^{\to-\infty}}{\underbrace{e^{-x}}_{\to+\infty}} \boxed{H} = \lim_{x\to-\infty} \frac{1}{\underbrace{-e^{-x}}_{\to-\infty}} = 0$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \underbrace{-x}_{\to-\infty}\ \underbrace{(1-\ln x)^2}_{\to+\infty} = -\infty \quad \text{pas AHD}$$

[1] $$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} -(1-\ln x)^2 = -\infty \quad \text{BPD dir } (Oy)$$

$$\forall x \in \mathbb{R}_0^-: f'(x) = \frac{xe^x(1-e^x) - [1+(x-1)e^x](-e^x)}{(1-e^x)^2}$$

$$= e^x \frac{x - xe^x + 1 + xe^x - e^x}{(1-e^x)^2}$$

[1] $$= \frac{\overbrace{x+1-e^x}^{=g(x)<0}}{\underbrace{e^{-x}(1-e^x)^2}_{>0}} < 0$$

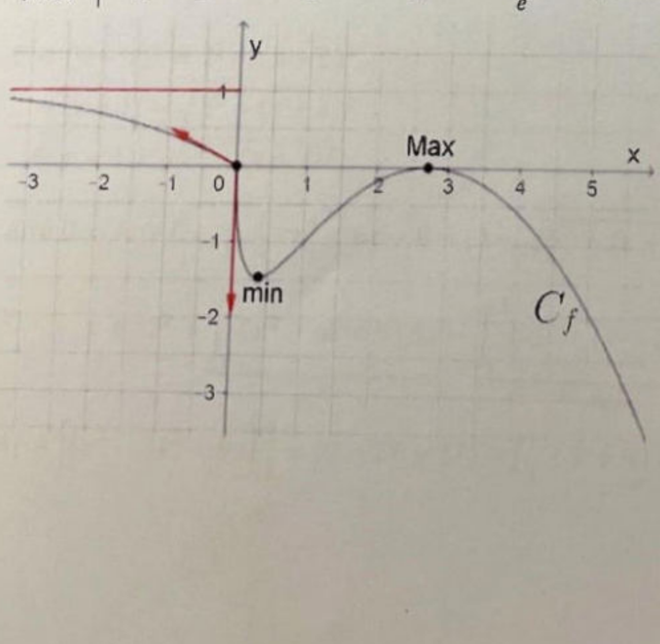
$$\forall x \in \mathbb{R}_0^+: f'(x) = -(1-\ln x)^2 - x\cdot 2(1-\ln x)\left(-\tfrac{1}{x}\right)$$
$$= -(1-\ln x)(1-\ln x - 2)$$
$$= (1-\ln x)(1+\ln x)$$
$$= 1-\ln^2 x$$

$$\forall x \in \mathbb{R}_0^+: f'(x) = 0 \Leftrightarrow \ln^2 x = 1 \Leftrightarrow \ln x = 1 \ \vee\ \ln x = -1 \Leftrightarrow x = e \ \vee\ x = \tfrac{1}{e}$$

[1] $$\forall x \in \mathbb{R}_0^+: f'(x) > 0 \Leftrightarrow \ln^2 x < 1 \Leftrightarrow -1 < \ln x < 1 \Leftrightarrow \tfrac{1}{e} < x < e$$

x	$-\infty$		0		$\frac{1}{e}$		e		$+\infty$
$f'(x)$		$-$	$-\frac{1}{2} \Vert -\infty$	$-$	0	$+$	0	$-$	
[1] $f(x)$	1	↘	0	↘	$-\frac{4}{e}$	↗	0	↘	$-\infty$

[2]



3)

$$\begin{aligned}
A(\lambda) &= \int_\lambda^e |f(x)|\,dx \\
&= \int_\lambda^e -f(x)\,dx \\
&= \int_\lambda^e x(1-\ln x)^2\,dx && \Bigg|\, IPP: \begin{matrix} u_1(x) = (1-\ln x)^2 & u_1'(x) = 2(1-\ln x)\left(-\frac{1}{x}\right) \\ v'(x) = x & v(x) = \frac{1}{2}x^2 \end{matrix} \\
&= \tfrac{1}{2}[x^2(1-\ln x)^2]_\lambda^e + \int_\lambda^e x(1-\ln x)\,dx && \Bigg|\, IPP: \begin{matrix} u_2(x) = 1-\ln x & u_1'(x) = -\frac{1}{x} \\ v'(x) = x & v(x) = \frac{1}{2}x^2 \end{matrix} \\
&= \tfrac{1}{2}(0-\lambda^2(1-\ln\lambda)^2) + \tfrac{1}{2}[x^2(1-\ln x)]_\lambda^e + \tfrac{1}{2}\int_\lambda^e x\,dx \\
&= -\tfrac{1}{2}\lambda^2(1-\ln\lambda)^2 + \tfrac{1}{2}\big(0-\lambda^2(1-\ln\lambda)\big) + \tfrac{1}{4}[x^2]_\lambda^e \\
&= \left[-\tfrac{1}{2}\lambda^2(1-\ln\lambda)^2 - \tfrac{1}{2}\lambda^2(1-\ln\lambda) + \tfrac{1}{4}(e^2-\lambda^2)\right]\ u.a.
\end{aligned}$$

4 $\qquad = \left[\frac{1}{4}e^2 - \frac{1}{2}\lambda^2\left(\frac{5}{2} - 3\ln\lambda + \ln^2\lambda\right)\right]\ u.a.$

4)

$$\begin{aligned}
\lim_{\lambda\to0^+} A(\lambda) &= \lim_{\lambda\to0^+}\left[\frac{1}{4}e^2 - \frac{1}{2}\lambda^2\left(\frac{5}{2} - 3\ln\lambda + \ln^2\lambda\right)\right] \\
&= \frac{1}{4}e^2 - \frac{5}{4}\underbrace{\lim_{\lambda\to0^+}\lambda^2}_{=0} + \frac{3}{2}\underbrace{\lim_{\lambda\to0^+}\lambda^2\ln\lambda}_{=0\ (CAP)} - \frac{1}{2}\underbrace{\lim_{\lambda\to0^+}\lambda^2\ln^2\lambda}_{=0\ (CAP)}
\end{aligned}$$

$$\text{CAP: } \lim_{\lambda\to0^+}\underbrace{\lambda^2}_{\to0}\underbrace{\ln\lambda}_{\to-\infty} = \lim_{\lambda\to0^+}\frac{\overbrace{\ln\lambda}^{\to-\infty}}{\underbrace{\lambda^{-2}}_{\to+\infty}}\ \boxed{H} = \lim_{\lambda\to0^+}\frac{\frac{1}{\lambda}}{-2\lambda^{-3}} = \lim_{\lambda\to0^+}\frac{\lambda^2}{-2} = 0$$

$$\text{CAP: } \lim_{\lambda\to0^+}\underbrace{\lambda^2}_{\to0}\underbrace{\ln^2\lambda}_{\to+\infty} = \lim_{\lambda\to0^+}\frac{\overbrace{\ln^2\lambda}^{\to+\infty}}{\underbrace{\lambda^{-2}}_{\to+\infty}}\ \boxed{H} = \lim_{\lambda\to0^+}\frac{2\ln\lambda\cdot\frac{1}{\lambda}}{-2\lambda^{-3}} = \lim_{\lambda\to0^+}\frac{\overbrace{\ln\lambda}^{\to-\infty}}{\underbrace{-\lambda^{-2}}_{\to-\infty}} = 0$$

2 $\qquad = \frac{1}{4}e^2\ u.a.$

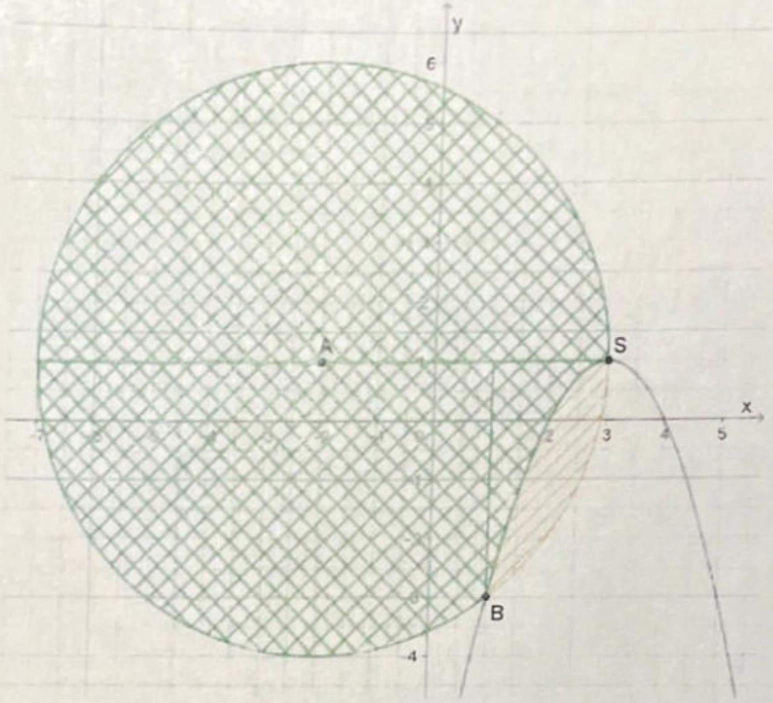
Question 5 (2 + 8 = 10 points)

1)

$C_f \equiv y = -x^2 + 6x - 8$ est une parabole concave dont le sommet S admet comme abscisse $-\frac{6}{2\cdot(-1)} = 3$ et comme ordonnée $f(3) = -3^2 + 6 \cdot 3 - 8 = 1$.

C_f passe par le point (4; 0).

1



D'après la figure, C_f et le cercle C ont deux points d'intersection : $S(3; 1)$ et $B(1; -3)$.

Le sommet S$\in C_f$ et $f(1) = -1^2 + 6 \cdot 1 - 8 = -3$ donc B$\in C_f$.

$C \equiv (x+2)^2 + (y-1)^2 = 5^2$

S$\in C$ car $(3+2)^2 + (1-1)^2 = 5^2 + 0^2 = 5^2$ et

1 B$\in C$ car $(1+2)^2 + (-3-1)^2 = 3^2 + 4^2 = 5^2$.

2)

$$C \equiv (y-1)^2 = 5^2 - (x+2)^2$$
$$\equiv |y-1| = \sqrt{5^2 - (x+2)^2}$$
$$\equiv y = 1 \pm \sqrt{5^2 - (x+2)^2}$$

1 Aire D = Aire $C - \int_1^3 \left(-x^2 + 6x - 8 - \left(1 - \sqrt{5^2 - (x+2)^2}\right)\right) dx$

$$= \pi \cdot 5^2 + \underbrace{\int_1^3 (x^2 - 6x + 9)\, dx}_{A_1} - \underbrace{\int_1^3 \sqrt{5^2 - (x+2)^2}\, dx}_{A_2}$$

1 $A_1 = \left[\frac{1}{3}x^3 - 3x^2 + 9x\right]_1^3 = (9 - 27 + 27) - \left(\frac{1}{3} - 3 + 9\right) = \frac{8}{3}$

$$A_2 = 5\int_1^3 \sqrt{1-\left(\frac{x+2}{5}\right)^2}\,dx \quad \left|\begin{array}{l}\frac{x+2}{5} = \cos t \Leftrightarrow x = -2+5\cos t\\ \Rightarrow dx = -5\sin t\; dt\end{array}\right. \quad \left|\begin{array}{l}x_1 = 1 \Rightarrow \cos t_1 = \frac{3}{5} \Rightarrow t_1 = \operatorname{Arc}\cos\frac{3}{5}\\ x_2 = 3 \Rightarrow \cos t_2 = 1 \Rightarrow t_2 = 0\end{array}\right.$$

$$= 5\int_{\operatorname{Arc}\cos\frac{3}{5}}^{0} \sqrt{1-\cos^2 t}\,(-5\sin t)\,dt$$

$$= -25\int_{\operatorname{Arc}\cos\frac{3}{5}}^{0} |\sin t|\sin t\; dt = 25\int_0^{\operatorname{Arc}\cos\frac{3}{5}} |\sin t|\sin t\; dt$$

$$= 25\int_0^{\operatorname{Arc}\cos\frac{3}{5}} \sin^2 t\; dt \quad \left(\text{car } \forall t \in \left[0;\operatorname{Arc}\cos\tfrac{3}{5}\right], \sin t \geq 0 \text{ et } |\sin t| = \sin t\right)$$

$$= \frac{25}{2}\int_0^{\operatorname{Arc}\cos\frac{3}{5}} (1-\cos 2t)\,dt$$

$$= \frac{25}{2}\left[t - \frac{1}{2}\sin 2t\right]_0^{\operatorname{Arc}\cos\frac{3}{5}}$$

$$= \frac{25}{2}\left(\operatorname{Arc}\cos\frac{3}{5} - \frac{1}{2}\sin\left(2\operatorname{Arc}\cos\frac{3}{5}\right) - 0\right)$$

$$\text{CAP : } \sin\left(2\operatorname{Arc}\cos\tfrac{3}{5}\right) = 2\sin\left(\operatorname{Arc}\cos\tfrac{3}{5}\right)\cos\left(\operatorname{Arc}\cos\tfrac{3}{5}\right) = 2\sqrt{1-\left(\tfrac{3}{5}\right)^2}\cdot\frac{3}{5} = \frac{6}{5}\sqrt{\frac{16}{25}} = \frac{24}{25}$$

$$= \frac{25}{2}\left(\operatorname{Arc}\cos\frac{3}{5} - \frac{1}{2}\cdot\frac{24}{25}\right)$$

5 $$= \frac{25}{2}\operatorname{Arc}\cos\frac{3}{5} - 6$$

$$\text{Aire } D = 25\pi + \frac{8}{3} - \frac{25}{2}\operatorname{Arc}\cos\frac{3}{5} + 6$$

$$= \left(25\pi + \frac{26}{3} - \frac{25}{2}\operatorname{Arc}\cos\frac{3}{5}\right)\; u.a.$$

1 $$\simeq 75{,}62\; u.a.$$